idea of a parachute leap, but do not at all support an evolution of flight by beating wings.

Exocœtus is the natural parallel of the aëroplane, which, it is hoped, will rise from and descend upon water with ease and perfect safety. The flying fish, however, frequently strikes a wave with one fin and is overturned, or strikes it with violence. It would be very interesting to know whether Belone does aid itself by its tail, and so is in some way a parallel to the hydroplane boat.

CYRIL CROSSLAND.

Dongonab, Port Sudan, Red Sea, March 24.

The Stinging Tree of Formosa.

WITH reference to the letter on the Stinging Tree of Formosa in NATURE of March 2, it would be interesting if your correspondent would throw light on the exact mechanism by which the sting in *Laportea pterostigma* and *L. crenulata* is produced. *L. crenulata* is locally abundant in some parts of India. The curious point is that the leaves are often glabrous. Moreover, the stinging effects are, apparently, sometimes experienced without actual contact with the plant. I was one day walking through the hot, steaming forests near the Tista River, in British Sikhim, with a friend. The Laportea was abundant, and we carefully avoided it. On our way home my friend was seized with the peculiar stinging sensations of the Laportea in several parts of his body. These lasted several days, and on the night immediately after being stung became so bad that he was unable to get any rest and became feverish.

On another occasion I had to cut a survey line through dense forest with an undergrowth of *L. crenulata*. The coolies avoided the leaves as much as possible, and cut the stems low. Some of them were stung on the body, but all were attacked in different degrees with sneezing, violent catarrh, and ultimately vertigo. I myself, although at some distance from the actual cutting operations, though I had to walk up the cut line, suffered to a less degree in the same way. Yet I have often dashed a leaf across the back of my hand with no ill effects! Sir J. Hooker and others have noted that the effects are worse at some times of the year than at others. The inflorescence, it should be noted, is covered with hairs, and I have only been able to account for the facts above described by supposing that it is these deciduous hairs of the inflorescence which get into the clothes and become inhaled when the tree is shaken.

H. H. HAINES.

Camp, Central Provinces, India, March 24.

Fundamental Notions in Vector Analysis.

I SHALL be much obliged if you will kindly permit me, through the columns of NATURE, to make some suggestions regarding fundamental conceptions in vector analysis, a subject which was vigorously discussed in this journal about twenty years ago (NATURE, vols. xliii., xliv., xlvii., xlviii., xlix.). The discussion showed that the slow progress of vector analysis was in a large measure due to the want of unanimity as to its fundamental notions and notations, and to an unfortunate aspect peculiar to it, viz., a strong conviction on the part of the advocate of any one of the various systems of vector analysis, that the other systems, if allowed to grow, will do more harm than good, while it may be noticed that in our ordinary scalar analysis, although several systems (*e.g.* Cartesian, polar, pedal, trilinear, &c.) exist side by side, there is no such feeling. My object now is to suggest a system which, while it aims at a reconcilement between the various systems, will contain the best features of each of these known systems.

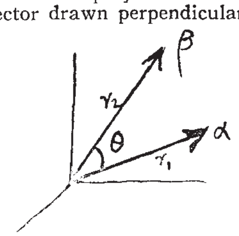
Dr. Knott (NATURE, vol. xlvii., p. 590) justifies the introduction of the quaternion as a fundamental conception by saying that it is only a generalisation to the case of vectors of the quotient (in the case of scalars) of two lengths. But a great objection is that the quaternion—a hybrid conception, in part a scalar and in part a vector—is not *by itself* capable of being defined in terms of the three fundamental entities, magnitude, direction, and position, as every *fundamental* conception ought to be. No such thing can, however, be said of the fundamental notions of the non-quaternionists, the scalar product and the vector product, which are defined in terms of only the fundamental notions of geometry and trigonometry. I may also repeat an argument of Prof. Gibbs (NATURE, vol. xlvii., p. 463) that the introduction of the scalar product and the vector product as fundamental conceptions will meet Prof. McAulay's observation (*Ph. Mg.*, vol. xxxiii. 1892, p. 477) that the arrest in the development of vector analysis is due to the circumstance that quaternions are "independent plants that require separate sowing and consequent careful tending." Besides, as is pointed out by Prof. Gibbs (NATURE, vol. xliii., p. 511), it is not desirable that the simpler conceptions should be expressed in terms of those which are by no means so. It is not sufficient to say, as has been argued (Heaviside, NATURE, vol. xlvii., p. 533), that vector analysis should have a purely vectorial basis; for that would only be a play of words.

Now, although the non-quaternionists thus avoid certain initial difficulties in presenting the subject, some of them, viz., Mr. Heaviside and Prof. Macfarlane, have made innovations which not only have no justification, but have created insuperable difficulties. We must have $\alpha^2=-1$, and we must recognise the versional character of the vector; the principles of vector algebra must differ as little as possible from the principles of scalar algebra, and we cannot be blind to the usual meaning of equations such as $ij=k$, &c., as was pointed out by Dr. Knott (NATURE, vol. xlviii., p. 148; vol. xlvii., p. 590). All these difficulties and others have arisen from an attempt to oust the conception of a quaternion, whether in the initial or at any later stage. So supreme is the contempt that Gibbs, while dealing with the theory of dyadics, regards $\alpha\beta+\lambda\mu+\gamma\nu$, a sum of expressions analogous to the quaternions, as indeterminate, merely symbolic, having physical meaning only when used as operator, although scalars and vectors are derived from it.

It is unfortunate that the advocates of vector analysis cannot work in harmony with one another, recognising superiority of each other in particular respects. Although Gibbs admits that the quaternionic method has advantages in certain cases, he would not tolerate its existence in the field of vector analysis, or rely upon it in places where he has found advantages.

With regard to the question of notations, I may refer to NATURE, vol. xlvii., p. 590, where Dr. Knott rightly says that the symbols used by the quaternionists for the scalar product and the vector product express at once and clearly the nature of the functions they represent, and that it is not proper to use the sign of ordinary multiplication in a case which does not admit of one of the factors being carried over to the other side as a divisor.

I shall now work out the successive stages of introducing the proposed system. We shall begin with the scalar product, $S\alpha\beta$, and the vector product, $V\alpha\beta$, defining the former as a quantity equal to minus the product of the length of one of the vectors, α, β, and the projection on it of the other, and the latter as a vector drawn perpendicular to the plane of the vectors, of a length equal to the area of the parallelogram determined by them, so that rotation round it from α to β through an angle less than 180° is positive. We see that we shall have



$$S\alpha\beta=S\beta\alpha,\quad V\alpha\beta=-V\beta\alpha.$$

Now if we take $\alpha=ix_1+jy_1+kz_1$

$\beta=ix_2+jy_2+kz_2$

we have,
$$\begin{aligned}
S\alpha\beta &= -T\alpha T\beta\cos\theta \\
&= -T\alpha\times\text{ projection of }T\beta\text{ on }\alpha \\
&= -\left[r_1.x_2.\frac{x_1}{r_1}+r_1.y_2.\frac{y_1}{r_1}+r_1.z_2.\frac{z_1}{r_1}\right] \\
&= -(x_1x_2+y_1y_2+z_1z_2
\end{aligned}$$

$V\alpha\beta=\ \ i$ (projection of area of parm. α, β on x plane)
$+j$ (projection of area of parm. α, β on y plane)
$+k$ (projection of area of parm. α, β on z plane)

$$=(y_1i_1z_2-y_2z_1)+j(z_1x_2-z_2x_1)+k(x_1y_2-x_2y$$

$$\begin{aligned}
\therefore\ S\alpha\beta+V\alpha\beta &= -(x_1x_2+y_1y_2+z_1z_2)+i(y_1z_2-y_2z_1)+ \\
&\quad j(z_1x_2-z_2x_1)+k(x,y_2-x_2y_1) \\
&= (ix_1+jy_1+kz_1)(ix_2+jy_2+kz_2) \\
&= \alpha\,\beta.
\end{aligned}$$

We thus arrive at an auxiliary, $\alpha\beta$, connected with the fundamental notions by the relation, $\alpha\beta = S\alpha\beta + V\alpha\beta$, an auxiliary the geometrical meaning of which will be seen below.

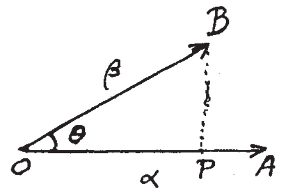
We then note the special case $\alpha^2 = S\alpha^2 = -(T\alpha)^2$ so that $\frac{1}{\alpha} = -\frac{\alpha}{T\alpha^2}$. With the help of this relation, we shall assign meanings to $S\frac{\beta}{\alpha}$, $V\frac{\beta}{\alpha}$:

$$S\frac{\beta}{\alpha} = S\frac{-\beta\alpha}{T\alpha^2} = \frac{-S\beta\alpha}{T\alpha^2}$$

$$V\frac{\beta}{\alpha} = V\frac{-\beta\alpha}{T\alpha^2} = \frac{-V\beta\alpha}{T\alpha^2}$$

From these we have
$$S\frac{\beta}{\alpha} + V\frac{\beta}{\alpha} = -\frac{1}{T\alpha^2}(S\beta\alpha + V\beta\alpha) = \frac{-\beta\alpha}{T\alpha^2} = \frac{\beta}{\alpha}$$

Apart from this, the following geometrical consideration will justify our introduction of the quotient and the conception of an operator:—



$$S\frac{\beta}{\alpha} + V\frac{\beta}{\alpha} = S\frac{-\beta\alpha}{T\alpha^2} + V\frac{-\beta\alpha}{T\alpha^2}$$
$$= \frac{T\beta.T\alpha\cos\theta}{T\alpha^2} + \epsilon.\frac{T\beta.T\alpha\sin\theta}{T\alpha^2}$$

where ϵ is an unit vector $+r$ to plane α, β

$$= \frac{T\beta\cos\theta}{T\alpha} + \epsilon.\frac{T\beta\sin\theta}{T\alpha}$$
$$= \frac{OP}{OA} + \frac{PB}{OA} = \frac{OB}{OA} = \frac{\beta}{\alpha}$$

We have *now*, but not earlier, the conception of our auxiliary, whether the product $\alpha\beta$ or the quotient $\frac{\beta}{\alpha}$, as an operator turning one vector into another, the former β^{-1} into α and the latter α into β; this would, as usual, justify our calling by their old name (quaternion) these auxiliaries which we have here *obtained from* our fundamental conceptions, the scalar product and the vector product. We may now proceed further and introduce into our system the conception of the axes and the angle of a quaternion. We may, and as a matter of fact shall, use the quaternion whenever we find it expedient, but we must not make it our fundamental notion.

In view of the diversity of opinion shown above, some modification and reconcilement on the lines suggested above or on some other lines are absolutely necessary, if the advocates of vector analysis are earnest in their desire to see it universally applied, and the Cartesian and other methods completely overthrown.

MAUMATHA NATH RAY.

Calcutta Mathematical Society, Senate House, March 2.

This method of approach to the quaternion vector analysis is practically that adopted by Prof. Joly in his "Manual of Quaternions." The method is unsatisfactory, because it makes too great a demand at the outset upon the learner's faith. Why should $S\alpha\beta$ be put equal to $-ab\cos\theta$? The answer is, of course, because that is the simplest way of getting a vector algebra applicable to Euclidean space, and at the same time associative in its vector products. But the existence of so many varieties of non-associative vector algebra shows how absolutely unimportant this latter consideration is to many who find vector analysis useful. In these varieties not only is there no explicit recognition of a quantity $\alpha\beta$, where α and β are vectors, but there is a perfect hatred of the mere suggestion of it as a quantity worthy of general discussion, except (be it noted) in the particular case in which α is perpendicular to β. Mr. Ray shows, by a simple Cartesian process, how easily we may arrive at the recognition of this product if we start with the geometrical definitions of Hamilton's $V\alpha\beta$ and $S\alpha\beta$. But the method is unconvincing to the man who prejudges the whole matter by barring out the quantity or symbolic form $\alpha\beta$ as being fundamentally foreign to any well-regulated system of vector analysis! If they would not listen to Hamilton, Tait, or Joly, will they listen to any other quaternionist, charm he never so wisely?

C. G. K.

An Abnormal Zebra.

IN reference to the note by Prof. Ridgeway on a photograph of a zebra, or boute-quagga, skin from the Athi Plains of British East Africa, published in NATURE of April 20, I write to say that a copy of the same photograph was received at this museum from Mr. Woosnam.

As I have mentioned in a note in *The Field* of April 22, Mr. Woosnam stated that there were only one or two of such abnormally marked animals running in a herd of *granti* at any one time. It is therefore clear that there is no ground for regarding the variation as of racial value. On this point Mr. O. Thomas, to whom the photograph was sent by Mr. Woosnam, is in complete accord with myself.

R. LYDEKKER.

British Museum (Natural History), Cromwell Road, London, S.W., April 24.

A Robin and his Young.

LAST summer a pair of robins built their nest in an old fish-basket that was hanging in a shed at the back of my house. All went well until the young birds were about a week old—then happened what appeared to me to be a catastrophe. My Aberdeen terrier pup "Bebe," who must have had some natural desire to catch the mother bird, managed one morning to make a meal of her.

Contrary to what I should have expected, the male bird kept close to his young family. Day by day I turned over part of the garden to supply him with a little help in his task. In due course he taught the whole of his young family to fly

I have made inquiries, but cannot find anyone who has had a similar experience, and wondered what your readers might know about such cases.

CHARLIE WOODS.

"Vectis," 2 Wellmeadow Road, Lewisham, S.E., April 18.

PROPOSALS FOR THE REFORM OF THE CALENDAR.

THE importance of a uniform and simple calendar is not a question which affords any ground for dispute. Whether regarded from the point of view of the chronologist, striving to evolve order out of regnal years and intercalary months, or from that of a business man in Cairo, transacting affairs with clients who adhere severally to the Moslem, the Coptic, the Hebrew, the Julian, and the Gregorian calendars, the diversity of system from time to time, from place to place, and between creed and creed, is an exasperating and unmixed misfortune. The New Year festival is celebrated by the motley races which go to make up the population of Singapore on dates which extend over several months. In Constantinople, until quite recently, even the division of the day was a source of grave inconvenience, since the day ended at local sunset. The persistence of such anomalies shows how hard is the way of the reformer. Tradition and religious scruple, and even the mere inertia of custom, are leagued against him. From the point of view of the whole world, a far greater advance would be made by any large step towards the adoption of one universal calendar than by making small theoretical improvements in a par-